

# Markov modeling and analysis of multi-stage manufacturing systems with remote quality information feedback<sup>☆</sup>



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## ABSTRACT

Modeling and analysis of multi-stage manufacturing systems (MMSs) for product quality propagation have attracted a great deal of attention recently. Due to cost and resources constraints, MMSs do not always have ubiquitous inspection, and MMSs with remote quality information feedback (RQIF, i.e., quality inspection operation is conducted at the end of the production line) are widely applied. This paper develops a Markov model to analyze quality propagation in MMSs with RQIF. Analytical expressions of the final product quality are derived and the monotonicity properties are investigated. A quality bottleneck identification method is explored based on the proposed Markov model. The results of case study demonstrate the effectiveness of the proposed model.

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## 1. Introduction

Most complex manufacturing systems involve a large number of stages. As workpieces move through these stages, the variations of product quality are usually introduced and propagated. The variations of the final product quality are the accumulation of variations from all stages. Therefore, it is significantly important to investigate the product quality propagation in multi-stage manufacturing systems (MMSs).

Modeling and analysis of MMSs for quality improvement have received intensive investigation. Recent quantitative modeling methods can be roughly classified as data-driven (applying statistical approaches based on measurement data) and analytical (applying physical approaches based on engineering knowledge) methods. Data-driven methods focus on investigating patterns in the massive historical quality dataset to model the relationships between the product quality and manufacturing systems and thus do not require a comprehensive prior knowledge of the systems. Some authors employ data-driven auto-regression (AR) models to describe the quality propagation (Agrawal, Lawless, & Mackay, 1999; Lawless, Mackay, & Robinson, 1999). The parameters of their AR(1) models are estimated based on product measurements.

Some authors develop data-driven modeling methods based on the analysis of the linear space spanned by the eigenvectors of the covariance matrix of the quality measurements (Jin & Zhou, 2006; Johnson & Wichern, 2002). Mondal, Maiti, and Ray (2013) combined statistical regression, Taylor series expansion and a variation model to investigate the robustness of MMSs.

Different from data-driven models, analytical models employ off-line analysis of MMSs based on fundamental physical laws. One of the most popular analytical models used for quality improvement is the state space model, first developed by Jin and Shi (1999) for two-dimensional assembly systems. This model directly links engineering knowledge of variation sources with product measurement data. Since then, it is further investigated in three-dimensional assembly systems (Camelio, Hu, & Ceglarek, 2004; Ding, Ceglarek, & Shi, 2002a; Huang, Lin, Bezdecny, Kong, & Ceglarek, 2007; Huang, Lin, Kong, & Ceglarek, 2007; Loose, Chen, & Zhou, 2009; Zhou, Qiang, & Zhou, 2012). However, these models could not be applied in the machining systems directly since the fundamental physical laws of quality propagation are quite different for assembly and machining systems (Du, Yao, Huang, & Wang, 2015). Therefore, some authors (Abellan-Nebot, Liu, Subirón, & Shi, 2012; Djurdjanovic & Ni, 2001, 2003, 2006; Du, Yao, & Huang, 2014, 2015; Huang & Shi, 2004a, 2004b; Huang, Shi, & Yuan, 2003; Loose, Zhou, & Ceglarek, 2007; Loose, Zhou, Zhou, & Ceglarek, 2010; Wang, Huang, & Katz, 2005; Zhou, Chen, & Shi, 2004; Zhou, Huang, & Shi, 2003) investigate the variation propagation for machining systems by applying the

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## Nomenclature

$M_i$	the $i$ th stage in MMSs	$\theta_i$	when the coming part is defective, the probability for $M_i$ to transit from state $d_i$ to state $g_i$
$M'_i$	the stage merged by the first $i$ stages in MMSs	$\lambda'_i$	quality failure probability for $i$ th stage in general Markov model
$g_i$	$M_i$ or $M'_i$ is producing a good product	$\mu'_i$	quality repair probability for $i$ th stage in general Markov model
$d_i$	$M_i$ or $M'_i$ is producing a defective product	$\lambda''_i$	quality failure probability without repair for $i$ th stage in quality flow model
$g_i g_{i+1}$	$M_i$ or $M'_i$ is producing a good product and $M_{i+1}$ is also producing a good product	$\nu''_i$	quality failure probability with repair for $i$ th stage in quality flow model
$g_i d_{i+1}$	$M_i$ or $M'_i$ is producing a good product and $M_{i+1}$ is producing a defective product	$\mu''_i$	quality repair probability for $i$ th stage in quality flow model
$d_i g_{i+1}$	$M_i$ or $M'_i$ is producing a defective product and $M_{i+1}$ is producing a good product	$X_{i,t}$	the matrix of state probabilities at time $t$ for the system with $i$ stages
$d_i d_{i+1}$	$M_i$ or $M'_i$ is producing a defective product and $M_{i+1}$ is also producing a defective product	$X_i$	the matrix of steady-state probabilities for the system with $i$ stages
$P$	the probability of the system in one certain steady state	$A_i$	the matrix of state transition probabilities for the system with $i$ stages
$\alpha_1$	the probability for $M_1$ to transit from state $g_1$ to state $d_1$	$S_{\gamma_k}$	the sensitivity of $P(g_k)$ with respect to $\gamma_k$
$\beta_1$	the probability for $M_1$ to transit from state $d_1$ to state $g_1$	$S_{\mu_k}$	the sensitivity of $P(g_k)$ with respect to $\mu_k$
$\alpha'_i$	the probability for $M'_i$ to transit from state $g_i$ to state $d_i$ ( $i \geq 2$ )	$S_{\eta_k}$	the sensitivity of $P(g_k)$ with respect to $\eta_k$
$\beta'_i$	the probability for $M'_i$ to transit from state $g_i$ to state $g_i$ ( $i \geq 2$ )	$S_{\theta_k}$	the sensitivity of $P(g_k)$ with respect to $\theta_k$
$\gamma_i$	when the coming part is good, the probability for $M_i$ to transit from state $g_i$ to state $d_i$		
$\mu_i$	when the coming part is good, the probability for $M_i$ to transit from state $d_i$ to state $g_i$		
$\eta_i$	when the coming part is defective, the probability for $M_i$ to transit from state $g_i$ to state $d_i$		

state space model. Detailed descriptions of existing research on the state space model are provided in a monograph (Shi, 2006) and a survey (Shi & Zhou, 2009). However, analysis of complex systems using the state space model based on physical laws is often intractable (Shi & Zhou, 2009), and such analysis either relies on complicated kinematics model of manufacturing systems, or is only applicable to deal with dimensional errors and the application area is limited (Ju, Li, Xiao, & Arinez, 2013; Ju, Li, Xiao, Huang, & Biller, 2014).

In another research line, Markov model has been widely used as analytical tool to investigate the interactions between manufacturing system design and product quality. Inman, Blumenfeld, Huang, and Li (2003) pointed out that product quality and manufacturing system design are tightly coupled. They reviewed the related literature and empirical evidence to show that manufacturing system design has a significant impact on product quality. Since then, the coupling between manufacturing system design and product quality has received more and more research attention. Kim and Gershwin (2005) developed a Markov model for machines with both quality and operational failures, and identified important differences between types of quality failures. Li and Huang (2007) applied a Markov model to evaluate quality performance and derived some closed expressions to calculate good part probability. Kim and Gershwin (2008) proposed analytical and computational methods using Markov model to evaluate three cases of long manufacturing lines with quality and operational failures. Li, Blumenfeld, and Marin (2008) investigated the impact of manufacturing system design on product quality through a case study at an automotive paint shop and introduce the notion of quality robustness. Wang, Li, Arinez, and Biller (2010) derived a closed formula to quantify the probability of producing a good part using a Markov chain model and investigated nonmonotonic properties of manufacturing systems. Colledani and Tolio (2011) proposed an analytical method for the joint design of quality and manufacturing parameters. Wang, Li, Arinez, and Biller (2012) introduced some indicators for identifying the quality improvability and bottleneck

sequence based on a Markov model. Ioannidis (2013) used a Markov model to investigate joint production and quality control in manufacturing systems with random demand. Wang, Li, Arinez, and Biller (2013) developed a Markov model to analyze product quality in manufacturing systems with batch productions and a notation of quality bottleneck transaction was introduced to describe the state transition that has the largest impact on quality. Zhao and Li (2014) developed Markov analytical models to characterize a furniture assembly system and lot size analysis and bottleneck analysis were carried out. The related literature are reviewed and new directions are provided by Inman, Blumenfeld, Huang, and Li (2013).

In spite of above effort, the current research work based on Markov models assume that each stage of a manufacturing system is independent, in other words, the product quality propagation does not considered in their systems. Ju et al. (2013, 2014) developed quality flow models to analyze product quality propagation and to identify the quality bottleneck for automotive paint system and battery manufacturing system respectively. The applicability of their models is demonstrated using case study. However, their quality flow models are based on assumption that the manufacturing system has ubiquitous inspection, namely, every stage has an inspection station. Due to cost and resources constraints in reality, it is not always possible to measure outputs and set up inspection station in every stage in a manufacturing system. A manufacturing system with remote quality information feedback (RQIF) is a representation of situations where most but not all operations are reliable in terms of quality and where the product defects are only inspected and identified at the end of the production line (see Fig. 1). This is not desirable, but it is often unavoidable and applied in reality (Kim & Gershwin, 2008; Montgomery, 2009).

Manufacturing systems with RQIF are often applied. Ding, Ceglarek, and Shi (2002b) described a multistage assembly system including three assembly stages and one measurement station at the end of the assembly line. Zantek, Wright, and Plante (2002,

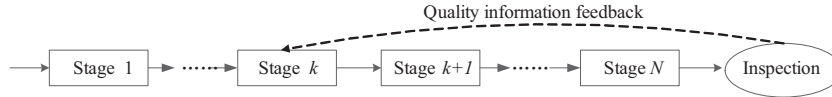


Fig. 1. A multi-stage manufacturing system with remote quality information feedback.

2006) described a circuit board assembly process consisting of nine stages, beginning at solder applicator and ending at manual solder. At the end of the assembly line, the board undergoes extensive testing and numerous product quality measurements. In semiconductor manufacturing, some defects are detected only at the circuit-testing stage, and this can only occur after chip fabrication is completed (Kim & Gershwin, 2008). In addition to the above mentioned examples, such systems can be found in many other manufacturing systems as well, such as engine manufacturing (Du & Xi, 2012) and aircraft horizontal stabilizer assembly (Du, Lv, & Xi, 2012; Du & Lv, 2013).

However, to best knowledge of the authors, there is no research work about developing Markov models to analyze the quality propagation in MMSs with RQIF. In MMSs with RQIF, there exist not only quality degradation but also quality correction. This means that because of RQIF, a defective product from upstream stages will not go out of the system until the last stage and they may be corrected by downstream stages. To reflect these characteristics and investigate such systems, in this paper, a Markov model is developed to analyze the quality bottleneck and provide some continuous quality improvement strategies.

The remainder of this paper is organized as follows. In Section 2, a Markov model is developed and analytical expressions of the final product quality are derived to analyze quality propagation in MMSs with RQIF. In Section 3, monotonicity properties are investigated. In Section 4, a quality bottleneck identification method is explored based on the developed Markov model. A case study is conducted to validate the proposed model in Section 5. Finally, the conclusion is given in Section 6.

## 2. Markov model

### 2.1. Assumptions

The assumptions of state transition, inspection and quality characteristics in MMSs with RQIF are described as follows.

- (1) The manufacturing system consists of  $n$  stages and one inspection station. The inspection station is at the end of the system.
- (2) The quality of the product manufactured by  $M_i$  ( $i \geq 2$ ) relies on both the state of  $M_i$  and the quality of the coming parts from  $M_{i-1}$ . There exist quality degradation and quality correction in the system. The product quality might get worse or better after a certain stage.
- (3) We only study the working or production period of the system. Machine breakdowns are not considered.

- (4) Define that stage  $M_i$  ( $i = 1, 2, \dots, n$ ) is in a good state  $g_i$  or a defective state  $d_i$  if it is producing a product with good quality or defective quality at time  $t$ .
- (5) The quality of the coming parts for  $M_i$  ( $i \geq 2$ ) at time  $(t + 1)$  depends on the state of  $M_{i-1}$  at time  $t$ . The states  $g_{i-1}$  and  $d_{i-1}$  for  $M_{i-1}$  at time  $t$  means good and defective coming parts for  $M_i$  at time  $(t + 1)$ , respectively.
- (6) The state of  $M_1$  is not affected by the state of  $M_2$ . When  $M_1$  is in good state  $g_1$ , it has probability  $\alpha_1$  to transit to defective state  $d_1$  and probability  $(1 - \alpha_1)$  to good state  $g_1$ . When  $M_1$  is in defective state  $d_1$ , it has probability  $\beta_1$  to transit to good state  $g_1$  and probability  $(1 - \beta_1)$  to defective state  $d_1$  (see Fig. 2(a)).

With good coming parts, when  $M_i$  ( $i \geq 2$ ) is in good state  $g_i$ , it has probability  $\gamma_i$  to transit to defective state  $d_i$  and probability  $(1 - \gamma_i)$  to good state  $g_i$ . When  $M_i$  is in defective state  $d_i$ , it has probability  $\mu_i$  to transit to good state  $g_i$  and probability  $(1 - \mu_i)$  to defective state  $d_i$  (see Fig. 2(b)).

With defective coming parts, when  $M_i$  ( $i \geq 2$ ) is in good state  $g_i$ , it has probability  $\eta_i$  to transit to defective state  $d_i$  and probability  $(1 - \eta_i)$  to good state  $g_i$ . When  $M_i$  is in defective state  $d_i$ , it has probability  $\theta_i$  to transit to good state  $g_i$  and probability  $(1 - \theta_i)$  to defective state  $d_i$  (see Fig. 2(c)).

We refer  $\alpha_i, \gamma_i, \eta_i$  ( $i \geq 2$ ) as quality failure probabilities and  $\beta_i, \mu_i, \theta_i$  ( $i \geq 2$ ) as quality repair probabilities. Similar to throughput analysis and in accordance with some works of quality analysis based on Markov model (Ju et al., 2013, 2014; Wang et al., 2010; Li & Huang, 2007), we assume that all these transition probabilities are constant. Actually in real manufacturing systems, machines have stable production periods during which the state transitions can be seen as stable.

The problem can be addressed as: Under the above assumptions, develop a proper method to evaluate and analyze the quality performance of MMSs with RQIF based on system parameters. Before deriving the model for multi-stage systems, the paper first develops a Markov model for a two-stage system, which will be extended for MMSs (the number of stages is more than two).

### 2.2. Two-stage case ( $n = 2$ )

For a manufacturing system with two stages, it has the following four probable quality states at time  $t$ : (1) state  $g_1g_2$  which means that both  $M_1$  and  $M_2$  are producing good products; (2) state  $d_1g_2$  which means that  $M_1$  is producing defective product while  $M_2$  is producing good one; (3) state  $g_1d_2$  which means that  $M_1$  is

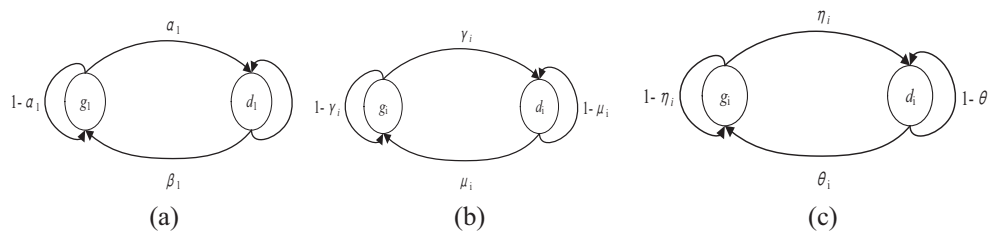


Fig. 2. State transition diagrams.

producing good product while  $M_2$  is producing defective one; (4) state  $d_1d_2$  which means that both  $M_1$  and  $M_2$  are producing defective products. The matrix of corresponding state probabilities at time  $t$  are denoted as

$$X_{2,t} = [P(g_1g_2, t) \quad P(d_1g_2, t) \quad P(g_1d_2, t) \quad P(d_1d_2, t)] \quad (1)$$

The system transits among these four states with certain probabilities. For example, transiting from state  $(g_1g_2, t)$  to  $(g_1g_2, t+1)$  means that: (1)  $M_1$  continues to produce good product with probability  $(1-\alpha_1)$  and it passes coming parts with good quality to  $M_2$ ; (2) with good coming parts,  $M_2$  maintains good state  $g_2$  with probability  $(1-\gamma_2)$ . So under assumptions (5) and (6), the probability that the system transits from state  $(g_1g_2, t)$  to  $(g_1g_2, t+1)$  could be the multiplication of the two probabilities:  $(1-\alpha_1)(1-\gamma_2)$ . Similarly, we can obtain the other transition probabilities. The matrix of state transition probabilities for a two-stage system is

$$A_2 = \begin{bmatrix} (1-\alpha_1)(1-\gamma_2) & \alpha_1(1-\gamma_2) & (1-\alpha_1)\gamma_2 & \alpha_1\gamma_2 \\ \beta_1(1-\eta_2) & (1-\beta_1)(1-\eta_2) & \beta_1\eta_2 & (1-\beta_1)\eta_2 \\ (1-\alpha_1)\mu_2 & \alpha_1\mu_2 & (1-\alpha_1)(1-\mu_2) & (1-\alpha_1)\mu_2 \\ \beta_1\theta_2 & (1-\beta_1)\theta_2 & \beta_1(1-\theta_2) & (1-\beta_1)(1-\theta_2) \end{bmatrix} \quad (2)$$

Taking state probability  $P(g_1g_2, t+1)$  for example, we have

$$\begin{aligned} P(g_1g_2, t+1) &= P(g_1g_2, t+1|g_1g_2, t) + P(g_1g_2, t+1|d_1g_2, t) \\ &\quad + P(g_1g_2, t+1|g_1d_2, t) + P(g_1g_2, t+1|d_1d_2, t) \\ &= (1-\alpha_1)(1-\gamma_2)P(g_1g_2, t) + \beta_1(1-\eta_2)P(d_1g_2, t) \\ &\quad + (1-\alpha_1)\mu_2P(g_1d_2, t) + \beta_1\theta_2P(d_1d_2, t) \end{aligned} \quad (3)$$

In terms of steady states, we denote  $X_2$  as the matrix of steady-state probabilities.

$$X_2 = [P(g_1g_2) \quad P(d_1g_2) \quad P(g_1d_2) \quad P(d_1d_2)] \quad (4)$$

And we have

$$\lim_{t \rightarrow \infty} P(g_1g_2, t) := P(g_1g_2) \quad (5)$$

$$\lim_{t \rightarrow \infty} P(d_1g_2, t) := P(d_1g_2) \quad (6)$$

$$\lim_{t \rightarrow \infty} P(g_1d_2, t) := P(g_1d_2) \quad (7)$$

$$\lim_{t \rightarrow \infty} P(d_1d_2, t) := P(d_1d_2) \quad (8)$$

It follows that

$$\begin{aligned} P(g_1g_2) &= (1-\alpha_1)(1-\gamma_2)P(g_1g_2) + \beta_1(1-\eta_2)P(d_1g_2) \\ &\quad + (1-\alpha_1)\mu_2P(g_1d_2) + \beta_1\theta_2P(d_1d_2) \end{aligned} \quad (9)$$

By calculating the following equations

$$X_2A_2 = X_2 \quad (10)$$

$$P(g_1g_2) + P(d_1g_2) + P(g_1d_2) + P(d_1d_2) = 1 \quad (11)$$

we can get the probabilities of each state that the system is in, respectively.

The final quality can be seen as the probability that  $M_2$  is in state  $g_2$  of producing product with good quality.  $P(g_2)$  characterizes the probability of producing good product of the system and we have

$$P(g_2) = P(g_1g_2) + P(d_1g_2) = \frac{A}{A+B} \quad (12)$$

where

$$\begin{aligned} A &= [\gamma_2(1-\alpha_1) - \beta_1\eta_2][\alpha_1\mu_2\beta_1\theta_2 - \mu_2\theta_2(1-\alpha_1)(1-\beta_1)] \\ &\quad + [1 - (1-\beta_1)(1-\eta_2) + \alpha_1(1-\gamma_2)]\{\beta_1\theta_2[1 - (1-\alpha_1)(1-\mu_2)] \\ &\quad + \beta_1\mu_2(1-\theta_2)(1-\alpha_1)\} + [1 - (1-\alpha_1)(1-\gamma_2) \\ &\quad + \beta_1(1-\eta_2)]\{\theta_2(1-\beta_1)[1 - (1-\alpha_1)(1-\mu_2)] \\ &\quad + \beta_1\alpha_1\mu_2(1-\theta_2)\} \end{aligned}$$

$$\begin{aligned} B &= [\beta_1\theta_2 - \mu_2(1-\alpha_1)]\{\gamma_2(1-\alpha_1)[1 - (1-\beta_1)(1-\eta_2)] \\ &\quad + \alpha_1\beta_1\eta_2(1-\gamma_2)\} - [\alpha_1\mu_2 - \theta_2(1-\beta_1)] \\ &\quad \times \{\beta_1\eta_2[1 - (1-\alpha_1)(1-\gamma_2)] + \gamma_2\beta_1(1-\alpha_1)(1-\eta_2)\} \\ &\quad + \{[1 - (1-\alpha_1)(1-\gamma_2)][1 - (1-\beta_1)(1-\eta_2)] \\ &\quad - \alpha_1\beta_1(1-\gamma_2)(1-\eta_2)\}[1 - (1-\alpha_1)(1-\mu_2) + \beta_1(1-\theta_2)] \end{aligned}$$

See Appendix A for proof of Eq. (12).

When we ignore the quality propagation between the two stages, namely, the quality of the product passing  $M_2$  only depends on the state of  $M_2$ , we have  $\gamma_2 = \eta_2, \mu_2 = \theta_2$ . Under these conditions, the probability of producing good product can be obtained

$$P(g_2) = \frac{A}{A+B} = \frac{\mu_2}{\gamma_2 + \mu_2} \quad (13)$$

where

$$\begin{aligned} A &= (\alpha_1 + \beta_1)^2\mu_2 + (\alpha_1 + \beta_1)(1-\alpha_1-\beta_1)\mu_2^2 \\ &\quad + (\alpha_1 + \beta_1)(1-\alpha_1-\beta_1)\gamma_2\mu_2 \end{aligned}$$

$$\begin{aligned} B &= (\alpha_1 + \beta_1)^2\gamma_2 + (\alpha_1 + \beta_1)(1-\alpha_1-\beta_1)\gamma_2^2 \\ &\quad + (\alpha_1 + \beta_1)(1-\alpha_1-\beta_1)\gamma_2\mu_2 \end{aligned}$$

This conclusion is consistent with that in work of Li and Huang (2007) which is about the product quality of one-product-one-stage system.

We consider a special Bernoulli case which is often employed in quality analysis (Kim & Gershwin, 2005; Wang, Li, Arinez, Biller, & Huang, 2008). In this case, the system reliability follows a Bernoulli distribution:  $\alpha_1 + \beta_1 = 1, \gamma_2 + \mu_2 = 1, \eta_2 + \theta_2 = 1$ . Under these conditions, we have

$$P(g_2) = 1 - \gamma_2 + \alpha_1(\gamma_2 - \eta_2) \quad (14)$$

where  $(\gamma_2 - \eta_2)$  can be seen as the impact that the quality of the coming parts has on the final quality. The larger the difference between  $\gamma_2$  and  $\eta_2$ , the larger the impact is.

### 2.3. Multi-stage case ( $n > 2$ )

In order to construct a general quality model for MMSs with RQIF, we first derive the model for three-stage systems and introduce the iteration method. For a three-stage system, the final quality equals to the probability with which  $M_3$  is producing good product. The state of  $M_3$  depends on both the state of itself in the previous time and the quality of the coming parts from  $M_2$  which is equivalent to the final quality of the two-stage system  $M_1-M_2$ . We can derive the quality of system  $M_1-M_2$  by applying the Markov model derived in Section 2.2. Therefore, the following procedure is presented to obtain the final quality of the three-stage system.

- (1) Merge  $M_1$  and  $M_2$  to one stage  $M'_2$ .
- (2) Obtain the final product quality of system  $M'_2-M_3$  by applying the derived model for the two-stage system.

For the two-stage system  $M'_2-M_3$ , we have six basic parameters  $\alpha'_2, \beta'_2, \gamma_3, \mu_3, \eta_3, \theta_3$ , among which  $\gamma_3, \mu_3, \eta_3$  and  $\theta_3$  are parameters of  $M_3, \alpha'_2$  and  $\beta'_2$  are parameters of the merged stage  $M'_2$ . The parameter  $\alpha'_2$  denotes the probability that  $M'_2$  transits from state

$g'_2$  to state  $d'_2$ , which is equivalent to the probability that the two-stage system transits from state  $g_1g_2$  or  $d_1g_2$  to state  $g_1d_2$  or  $d_1d_2$ . As a result,

$$\alpha'_2 = \frac{P(g_1g_2)\gamma_2 + P(d_1g_2)\eta_2}{P(g_1g_2) + P(d_1g_2)} \quad (15)$$

Similarly

$$\beta'_2 = \frac{P(g_1d_2)\mu_2 + P(d_1d_2)\theta_2}{P(g_1d_2) + P(d_1d_2)} \quad (16)$$

For a multi-stage system, a general iterative procedure is illustrated in Fig. 3 and the main steps are presented as follows.

- (1) Merge  $M_1$  and  $M_2$  to one stage  $M'_2$ , and model the quality of the new two-stage system  $M'_2-M_3$ .
- (2) Merge  $M'_2$  and  $M_3$  to one stage  $M'_3$ , and model the quality of the new two-stage system  $M'_3-M_4$ .
- (3) Continue this iteration process until the first  $(n - 1)$  stages are merged to one stage  $M'_{n-1}$ , and model the quality of the final two-stage system  $M'_{n-1}-M_n$ .

During the iterative process of  $n$ -stage system, any two-stage system  $M'_i-M_{i+1}$  has six basic parameters. They are  $\gamma_{i+1}, \mu_{i+1}, \eta_{i+1}, \theta_{i+1}$  reflecting the characteristics of  $M_{i+1}$  and  $\alpha'_i, \beta'_i$  reflecting the characteristics of  $M'_i$ . Also, we have

$$\alpha'_i = \frac{P(g_{i-1}g_i)\gamma_i + P(d_{i-1}g_i)\eta_i}{P(g_{i-1}g_i) + P(d_{i-1}g_i)} \quad (17)$$

$$\beta'_i = \frac{P(g_{i-1}d_i)\mu_i + P(d_{i-1}d_i)\theta_i}{P(g_{i-1}d_i) + P(d_{i-1}d_i)} \quad (18)$$

The matrix of state transition probabilities is

$$A_{i+1} = \begin{bmatrix} (1 - \alpha'_i)(1 - \gamma_{i+1}) & \alpha'_i(1 - \gamma_{i+1}) & (1 - \alpha'_i)\gamma_i & \alpha'_i\gamma_{i+1} \\ \beta'_i(1 - \eta_{i+1}) & (1 - \beta'_i)(1 - \eta_{i+1}) & \beta'_i\eta_{i+1} & (1 - \beta'_i)\eta_{i+1} \\ (1 - \alpha'_i)\mu_{i+1} & \alpha'_i\mu_{i+1} & (1 - \alpha'_i)(1 - \mu_{i+1}) & (1 - \alpha'_i)\mu_{i+1} \\ \beta'_i\theta_{i+1} & (1 - \beta'_i)\theta_{i+1} & \beta'_i(1 - \theta_{i+1}) & (1 - \beta'_i)(1 - \theta_{i+1}) \end{bmatrix} \quad (19)$$

The matrix of the steady-state probabilities is

$$X_{i+1} = [P(g_i g_{i+1}) \quad P(d_i g_{i+1}) \quad P(g_i d_{i+1}) \quad P(d_i d_{i+1})] \quad (20)$$

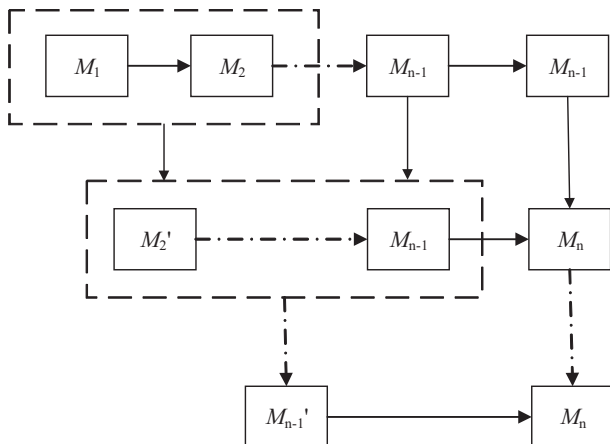


Fig. 3. Iterative procedure for multi-stage systems.

According to Markov model, we obtain

$$X_{i+1}A_{i+1} = X_{i+1} \quad (21)$$

$$P(g_i g_{i+1}) + P(d_i g_{i+1}) + P(g_i d_{i+1}) + P(d_i d_{i+1}) = 1 \quad (22)$$

The final probability of producing good product for the merged two-stage system is

$$P(g_{i+1}) = P(g_i g_{i+1}) + P(d_i g_{i+1}) \quad (23)$$

Therefore, to analyze the quality performance of MMSs, the basic idea is that: first merge the multiple stages into two stages with iteration method and then apply the Markov model developed in Section 2.2.

#### 2.4. Model accuracy investigation

In the multi-stage case, the iteration method is used to derive the model. To investigate the accuracy of this method, extensive numerical experiments have been carried out by randomly selecting the parameters of MMSs in a reasonable range of values.

In our model, the value ranges of quality failure probabilities and quality repair probabilities generally are  $[0, 1]$ . In the numerical experiments, based on actual production conditions, we narrow down the value ranges according to some works (Wang et al., 2010, 2013) and assume that

- (1) Quality failure probabilities with good coming parts are relatively small,  $\alpha_1 \in [0, 0.3]$  and  $\gamma_i \in [0, 0.3], (i \geq 2)$ .
- (2) Quality repair probabilities with good coming parts have large values  $\beta_1 \in [0.7, 1]$  and  $\mu_i \in [0.7, 1], (i \geq 2)$ .
- (3) Quality failure probabilities and quality repair probabilities with defective coming parts keep in the range of  $[0, 1], \eta_i \in [0, 1], \theta_i \in [0, 1]$ .

In accuracy investigation, the number of stages is from 2 to 10. More than 10,000 numerical experiments are carried out by randomly and uniformly generating parameters  $\alpha_1, \beta_1, \gamma_i, \mu_i, \eta_i, \theta_i$  within their value ranges based on the assumptions. We generate random numbers between 0 and 1 and zero denotes the state of producing defective product while one denotes the state of producing good product. Then we have the simulated probability of producing good product for the system. The difference between the results calculated from the developed model and from the simulation illustrates the accuracy of the model. The mean and variance of the difference with 2–10 processing stages are shown in Fig. 4(a) and (b), respectively.

From the above analysis, we can see that when the number of stages increases from two to ten, the mean error of the model compared with the simulation maintains in the range of  $\pm 0.2\%$  and the variance is less than  $1.5 \times 10^{-4}$ . As the production processes are stochastic, the differences between the model and the simulation are randomly distributed within their ranges. Therefore, the iteration method has sufficient accuracy assurance and can be used in the quality analysis of MMSs.

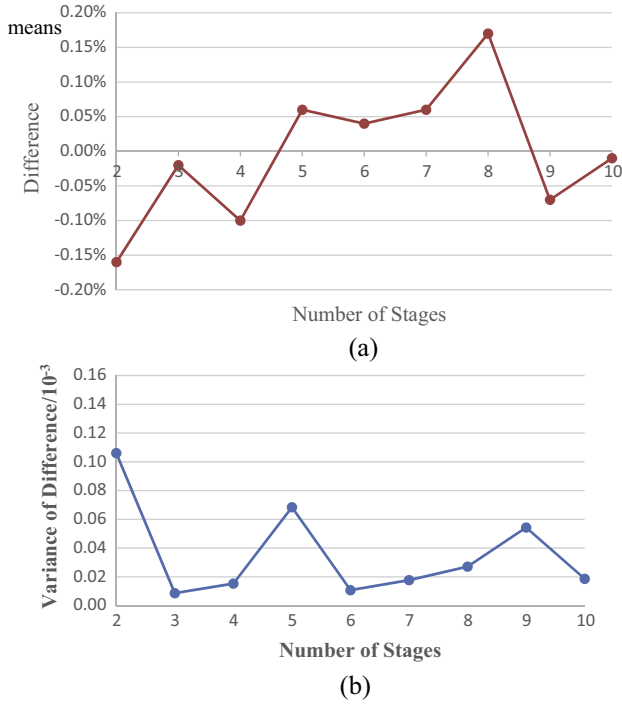


Fig. 4. Means and variance of difference between the model and the simulation.

### 2.5. Comparison with the state-of-the-art

Recent research that uses transition probabilities to study the quality propagation in MMSs is rare. After the authors making every effort to search, there is only one related paper Ju et al. (2014). Here we make a detailed comparison between the method proposed in this paper and the one in Ju et al. (2014) respectively.

Firstly, the two methods focus on different kinds of MMSs. In Ju et al. (2014), the assembly line has inspection stations and repair stations after each stage. In this paper, the MMSs have only one inspection station at the last stage and no repair stations exist. The most significant difference between these two kinds of systems is the ways of quality propagation. For systems in Ju et al. (2014), the coming parts for each stage are all with good quality, and the quality corrections by the system itself are not considered. But for the system with RQIF, the coming parts may be good or defective for each stage before being processed and there exist both quality degradation and quality correction. Actually in real manufacturing systems, there do exist the condition where a part with dissatisfactory quality becomes good after being processed. Taking a hole with dimensional requirement  $10_{-0.02}^{+0.02}$  (unit: mm) for example, when after rough machining, its dimension is 9.7 (mm) which is dissatisfactory, then it can be corrected to  $10_{-0.02}^{+0.02}$  (mm) by the downstream finish stage.

Secondly, the proposed methods are different although they're both based on transition probabilities. In Ju et al. (2014), they assume the probabilities of initial states  $P(g_0)$  and  $P(r_0)$  to be known and the final quality  $P(g_k)$  and  $P(r_k)$  are calculated by multiplying the transition probabilities by  $P(g_0)$  and  $P(r_0)$ . The manufacturing systems involve external interference. However, the MMSs in this paper have no repair station and the inspection station is also remote, so the whole manufacturing system is stochastic. Therefore, a Markov model is proposed to describe the characteristics of this kind of system.

In conclusion, the methods proposed by Ju et al. (2014) and this paper are quite different as their target systems quite differ from each other, although they are both based on transition probabilities. They are meant for different kinds of systems.

## 3. Analysis of monotonic properties

In this section, based on the above developed model, we will investigate the monotonic properties of the final quality. Intuitively, monotonic properties are thought to be expected. For example, we may expect that any increase in quality repair probabilities or any decrease in quality failure probabilities can lead to the improvement of final quality. However, if such monotonic properties do not hold, we will fail to obtain the quality improvement based on them. Therefore, studying the monotonic properties is of great importance for quality improvement.

Although we hold the view that the monotonic properties in terms of quality failure probabilities and quality repair probabilities are expected, numerical experiments suggest that these properties do not hold all the time. We will take the Bernoulli case of two-stage systems and equal-stage Bernoulli case of three-stage systems for example to illustrate the indeterminacy of the monotonic properties in MMSs.

### 3.1. Two-stage systems under Bernoulli case

According to Eq. (14), calculate the partial derivatives of  $\alpha_1, \gamma_2$  and  $\eta_2$  respectively

$$\frac{\partial P(g_2)}{\partial \alpha_1} = \gamma_2 - \eta_2 \quad (24)$$

$$\frac{\partial P(g_2)}{\partial \gamma_2} = \alpha_1 - 1 \quad (25)$$

$$\frac{\partial P(g_2)}{\partial \eta_2} = -\alpha_1 \quad (26)$$

It's obvious that the monotonic properties with respect to  $\gamma_2$  and  $\eta_2$  are determinate. The final quality will decrease with respect to  $\gamma_2$  and  $\eta_2$  as  $\alpha_1 - 1 \leq 0$  and  $-\alpha_1 \leq 0$ . However, the monotonicity with respect to  $\alpha_1$  depends on the difference between  $\gamma_2$  and  $\eta_2$ . When  $\gamma_2$  is larger than  $\eta_2$ , the final quality will increase with respect to  $\alpha_1$ , otherwise, it will decrease. This is out of our general expectations. This means that the difference between  $\gamma_2$  and  $\eta_2$  not only reflects the impact that the quality of the coming parts has on final quality, but also plays a determinate role in the way how parameters of  $M_1$  influence the final quality.

For a two-stage system, the monotonic properties with respect to quality failure probabilities and quality repair probabilities of the second stage are just as what we expected. However the monotonic properties with respect to the probabilities of the first stage depend on the value of the quality failure probabilities of  $M_2$  with good or defective coming parts.

### 3.2. Three-stage systems under equal-stage Bernoulli case

In a three-stage system, for simplification, besides Bernoulli conditions, we assume that transition probabilities for  $M_2$  and  $M_3$  are identical. Under this equal-stage Bernoulli case, we have  $\alpha_1 + \beta_1 = 1, \gamma_2 + \mu_2 = 1, \eta_2 + \theta_2 = 1, \gamma_3 = \gamma_2, \mu_3 = \mu_2, \eta_3 = \eta_2, \theta_3 = \theta_2$ . According to Eqs. (21)–(23), the final probability of producing good product for the system can be obtained as

$$\begin{aligned} P(g_3) &= 1 - \alpha_1 \eta_2^2 - \gamma_2 \eta_2 - \alpha_1 \gamma_2^2 - \gamma_2 + 2\alpha_1 \eta_2 \gamma_2 + \gamma_2^2 \\ &= 1 - \alpha_1 (\eta_2 - \gamma_2)^2 - \gamma_2 (\eta_2 - \gamma_2) - \gamma_2 \end{aligned} \quad (27)$$

Calculate the partial derivatives in terms of  $\alpha_1, \gamma_2$  and  $\eta_2$  respectively,

$$\frac{\partial P(g_3)}{\partial \alpha_1} = -(\eta_2 - \gamma_2)^2 \quad (28)$$

$$\frac{\partial P(g_3)}{\partial \gamma_2} = 2(\alpha_1 - 1)(\eta_2 - \gamma_2) + \eta_2 - 1 \quad (29)$$

$$\frac{\partial P(g_3)}{\partial \eta_2} = -2\alpha_1(\eta_2 - \gamma_2) - \gamma_2 \quad (30)$$

Eqs. (28)–(30) show that the monotonic properties with respect to  $\alpha_1$  are determinate and the decrease in  $\alpha_1$  will lead to quality improvement. But the monotonic properties with respect to  $\gamma_2$  and  $\eta_2$  depend on the value of  $\alpha_1$ ,  $\gamma_2$  and  $\eta_2$ .

The two examples verify that the monotonic properties of the final quality with respect to quality failure probabilities and quality repair probabilities are not always expected. But they still provide some insights about the quality of MMSs.

- (1) Theorem 1: For any two-stage system  $M'_i - M_{i+1}$  ( $i \geq 2$ ), the monotonic properties of  $P(g_{i+1})$  with respect to the second stage  $M_{i+1}$ 's parameters:  $\gamma_i, \mu_i, \eta_i, \theta_i$  are determinate. Reducing the quality failure probabilities  $\gamma_i$  and  $\eta_i$  or increasing the quality repair probabilities  $\mu_i$  and  $\theta_i$  will lead to the improvement of  $P(g_{i+1})$ .
- (2) As monotonic properties of the final quality  $P(g_n)$  with respect to certain parameters are not always the same as what we expect, we can first gain the basic parameters of the system and then use the models developed in this paper to do specific monotonic analysis.

#### 4. Bottleneck analysis

Bottleneck identification and mitigation have been viewed as one of the most important issues in quality analysis of manufacturing systems. It belongs to the theory of constraints (TOC) developed by Eli Goldratt in 1988 (Goldratt, 1988) and has been applied to many fields such as production, logistics, and supply chain (Zeynep, Noyan, & Ozalp, 2014). It is a methodology for identifying the most important limiting factor. The concept of TOC can be summarized as two important aspects (Rahman, 1998). (1) Every system must have at least one constraint limiting a system from achieving higher performance versus its goal. (2) The existence of constraints represents opportunities for improvement.

Based on TOC there must exist at least one stage or one parameter that limits the MMS achieving its goal of higher quality. Here we refer to it as quality bottleneck. And it provides some chances for the quality improvement of MMSs.

Here we define the stage after which the quality has the largest possible decline as the quality bottleneck stage. Let  $\Delta P(g_i)$  denote the quality change after stage  $M_i$  and we have

$$\Delta P(g_i) = P(g_i) - P(g_{i-1}) \quad (i \geq 2) \quad (31)$$

$$\Delta P(g_1) = P(g_1) - P(g_2) \quad (i = 1) \quad (32)$$

Then the stage with the most negative value of  $\Delta P(g_i)$  will be the quality bottleneck stage. As the bottleneck stage has been found, for quality improvement, we should consider that if only one parameter of the bottleneck stage  $M_i$  can be changed, which one will bring the largest quality improvement to  $P(g_i)$ . We define the parameter which has the largest effect on  $P(g_i)$  as the quality bottleneck parameter for  $M_i$ . When  $i \geq 2$ , this procedure can be seen as sensitivity analysis of  $P(g_i)$  with respect to  $\gamma_i, \mu_i, \eta_i$  and  $\theta_i$ . And when  $i = 1$ , it would be sensitivity analysis of  $P(g_1)$  with respect to  $\alpha_1$  and  $\beta_1$ .

For any two-stage system  $M'_{i-1} - M_i$  ( $i \geq 2$ ), assume that the existing parameters for  $M_i$  are  $\gamma_i, \mu_i, \eta_i, \theta_i$ , and the probability of producing good product for  $M_i$  is  $P(g_i)$ . Change only one parameter and remain the others unchanged. Accordingly, the changed parameters and probabilities are  $\gamma'_i, \mu'_i, \eta'_i, \theta'_i$  and

$P_{\gamma_i}(g_i), P_{\mu_i}(g_i), P_{\eta_i}(g_i), P_{\theta_i}(g_i)$ , respectively. Then the sensitivity of  $P(g_i)$  with respect to  $\gamma_i, \mu_i, \eta_i, \theta_i$  could be written as

$$S_{\gamma_i} = \frac{|P_{\gamma_i}(g_i) - P(g_i)|/P(g_i)}{|\gamma'_i - \gamma_i|/\gamma_i} \quad (33)$$

$$S_{\mu_i} = \frac{|P_{\mu_i}(g_i) - P(g_i)|/P(g_i)}{|\mu'_i - \mu_i|/\mu_i} \quad (34)$$

$$S_{\eta_i} = \frac{|P_{\eta_i}(g_i) - P(g_i)|/P(g_i)}{|\eta'_i - \eta_i|/\eta_i} \quad (35)$$

$$S_{\theta_i} = \frac{|P_{\theta_i}(g_i) - P(g_i)|/P(g_i)}{|\theta'_i - \theta_i|/\theta_i} \quad (36)$$

Specially, when  $i = 1$ , the sensitivity analysis of  $P(g_1)$  with respect to  $\alpha_1$  and  $\beta_1$  is needed. According to the work of Li and Huang (2007),

$$P(g_1) = \frac{\beta_1}{\alpha_1 + \beta_1} \quad (37)$$

Assume that the changed parameters are  $\alpha'_1, \beta'_1$  and  $P_{\alpha_1}(g_1), P_{\beta_1}(g_1)$ . Then the sensitivity of  $P(g_1)$  with respect to  $\alpha_1$  and  $\beta_1$  would be

$$S_{\alpha_1} = \frac{|P_{\alpha_1}(g_1) - P(g_1)|/P(g_1)}{|\alpha'_1 - \alpha_1|/\alpha_1} \quad (38)$$

$$S_{\beta_1} = \frac{|P_{\beta_1}(g_1) - P(g_1)|/P(g_1)}{|\beta'_1 - \beta_1|/\beta_1} \quad (39)$$

Parameter with  $\max(S_{\gamma_i}, S_{\mu_i}, S_{\eta_i}, S_{\theta_i})$  for  $i \geq 2$  or  $\max(S_{\alpha_1}, S_{\beta_1})$  for  $i = 1$  is the quality bottleneck parameter and its improvement will bring maximum benefit to the quality of product at the quality bottleneck stage.

#### 5. Case study

To validate the effectiveness of the proposed model and illustrate the method of quality bottleneck identification, a case study has been carried out at the manufacturing line of valve shell. To ensure the confidentiality of the data, all the parameters introduced below have been modified and only used for illustration.

##### 5.1. Experimental setup

The raw valve shell, the three-dimensional (3D) model and the profile of the valve from the axes of side A/B/C/D are respectively shown in Fig. 5(a)–(c). The experimental setup of manufacturing process is described as Fig. 6, the CXT-420 turning machines are applied, and the manufacturing process is roughly composed of five stages, named OP10–OP50 (see Fig. 7 and Table 1).

The relationships among these five stages and the ways how they are dependent have been discussed in detail in the work of Du et al. (2015). Here we only take some examples to show their connections briefly. The end face of the shell is machined in process OP10 and its variation of flatness will affect the clamping accuracy in processes OP20 and OP30. And the quality of the hole which is first bored in OP10 can be corrected or deteriorated in OP50.

The transition probabilities of this five-stage system can be estimated from historical processing data. We first keep records of the product quality before and after each stage and mark them as “good” or “defective”. For a certain part  $j$  which is processed by  $M_{i-1}$  ( $i \geq 2$ ), it can be either good or defective as coming parts for

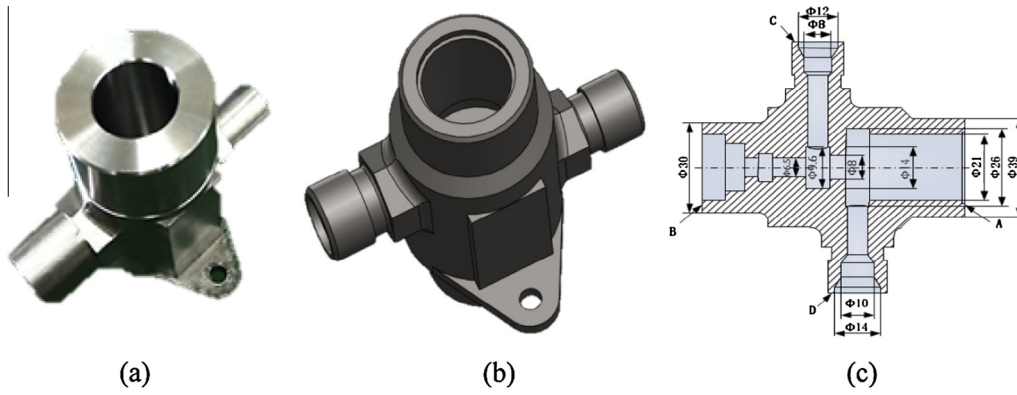


Fig. 5. Valve shell.

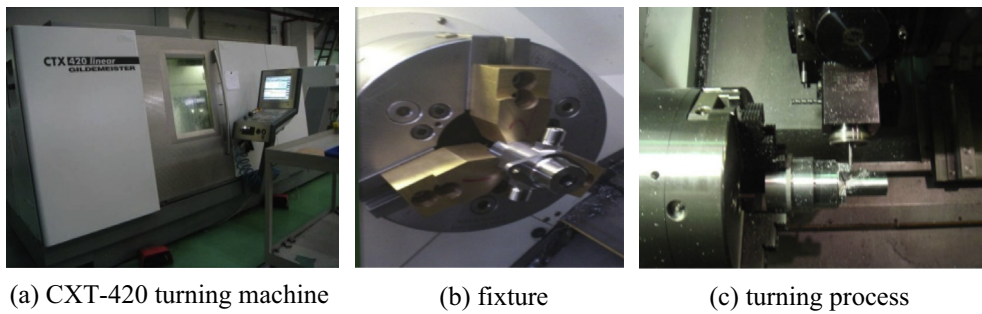


Fig. 6. Experimental setup of manufacturing process.

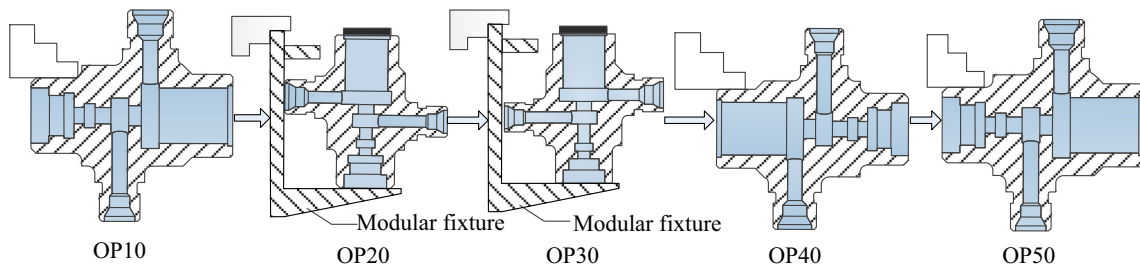


Fig. 7. Manufacturing process of valve shell.

the following stage  $M_i$ . And it may be processed after a good part or a defective one in  $M_i$ . After processing, in terms of a good or defective coming part, there exist four possible statuses for part  $j$  at  $M_i$ , respectively.

- (1) The previous part ( $j - 1$ ), processed by  $M_i$ , is good and  $j$  is also good.
- (2) The previous part ( $j - 1$ ), processed by  $M_i$ , is good but  $j$  is defective.
- (3) The previous part ( $j - 1$ ), processed by  $M_i$ , is defective but  $j$  is good.
- (4) The previous part ( $j - 1$ ), processed by  $M_i$ , is defective and  $j$  is also defective.

When coming parts are good, the proportion of statue (2) represents transition probability  $\alpha_1$  of  $M_1$  or  $\gamma_i$  of  $M_i$  ( $i \geq 2$ ). And the proportion of statue (3) equals to  $\beta_1$  of  $M_1$  or  $\mu_i$  of  $M_i$  ( $i \geq 2$ ). When coming parts are defective, proportions of (2) and (3) would be taken as  $\eta_i$  and  $\theta_i$ , respectively.

Table 1  
The manufacturing process with five stages.

Stage	Clamping method	Machining feature
OP10	B, 4-jaws chuck	Hole $\Phi 21$ Excircle $\Phi 39$
OP20	B downward, modular fixture (processing D)	Hole $\Phi 14$ Hole $\Phi 10$
OP30	B downward, modular fixture (processing C)	Hole $\Phi 12$ Hole $\Phi 8$
OP40	A, 4-jaws chuck	Slot $\Phi 9.6$ Hole $\Phi 6.5$ Excircle $\Phi 30$
OP50	B, 4-jaws chuck	Slot $\Phi 26$ Slot $\Phi 14$ Hole $\Phi 8$



5.2. Results and analysis

The characteristics of the five dependent stages on the factory floor are presented in Fig. 8(a)–(e) in the form of quality failure probabilities and quality repair probabilities. All the probabilities are based on historical processing data.

As we can see,  $\alpha_1 = 0.05, \beta_1 = 0.9, \gamma_i = [0.05, 0.1, 0.05, 0.05], \mu_i = [0.8, 0.8, 0.9, 0.9], \eta_i = [0.5, 0.5, 0.4, 0.5], \theta_i = [0.4, 0.3, 0.2, 0.4]$ . According to the proposed model, the quality of the product through the MMSs is obtained and shown in Fig. 9.

The final probability of producing good product estimated from the model is 89.07% and the actual final quality based on historical data is 89.43%, and the difference is 0.36%. The result proves the effectiveness and practicability of the model.

The quality changes after each stage are calculated as follows:

$$\Delta P(g_1) = P(g_1) - P(g_2) = 3.38\% \tag{40}$$

$$\Delta P(g_2) = P(g_2) - P(g_1) = -3.38\% \tag{41}$$

$$\Delta P(g_3) = P(g_3) - P(g_2) = -6.51\% \tag{42}$$

$$\Delta P(g_4) = P(g_4) - P(g_3) = 3.40\% \tag{43}$$

$$\Delta P(g_5) = P(g_5) - P(g_4) = 0.82\% \tag{44}$$

As  $\Delta P(g_3)$  has the most negative value, stage OP30 is the quality bottleneck stage. Then the monotonicity and sensitivity analysis is needed to find out how the changes of parameters affect quality  $P(g_3)$  and which one brings the largest improvement to OP30. In this case,  $\gamma_3, \mu_3, \eta_3,$  and  $\theta_3$  are increased or decreased by given percentages. Specifically, they are modified by  $\pm 10\%, \pm 15\%$  and  $\pm 20\%$ , respectively. The quality  $P(g_3)$  corresponding to these changes are shown in Fig. 10. The sensitivities with respect to the four parameters at 10% are  $S_{\gamma_3} = 10.39\%, S_{\mu_3} = 12.98\%, S_{\eta_3} = 4.6\%, S_{\theta_3} = 0.71\%$ , respectively.

We have made some changes of three parameters in the system including modifying the cutter compensation of OP30, changing cutting speed and feed speed. However, because of the complexity of data collection and the limitation of actual production conditions, the changes of other parameters cannot be obtained from real data and are based on numerical simulation. The quality changes are consistent with the results of the proposed model.

According to the analysis, some findings can be obtained:

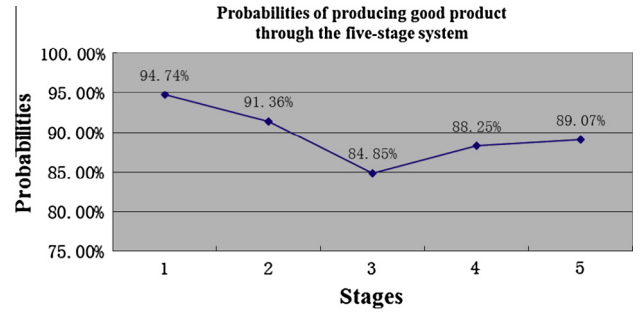


Fig. 9. Estimated probabilities of producing good product through the five-stage system.

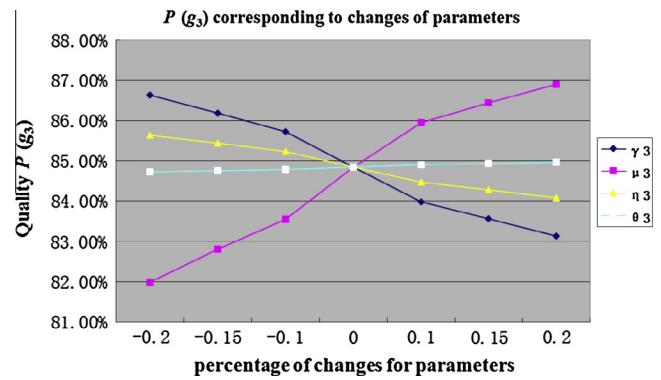


Fig. 10.  $P(g_3)$  changes corresponding to changes of parameters.

- (1) The monotonic properties of  $P(g_3)$  hold, which is in accordance with Theorem 1. The quality will be improved when quality failure probabilities of  $M_3$  decrease or the quality repair probabilities of  $M_3$  increase.
- (2) With the proposed model, the quality bottleneck stage and parameter can be identified. In this case, OP30 is the quality bottleneck stage because quality has the largest decline at it. In sensitivity analysis of the quality bottleneck stage, we have  $S_{\mu_3} > S_{\gamma_3} > S_{\eta_3} > S_{\theta_3}$ . As a result, in terms of stageOP30, the quality is pretty sensitive to the quality

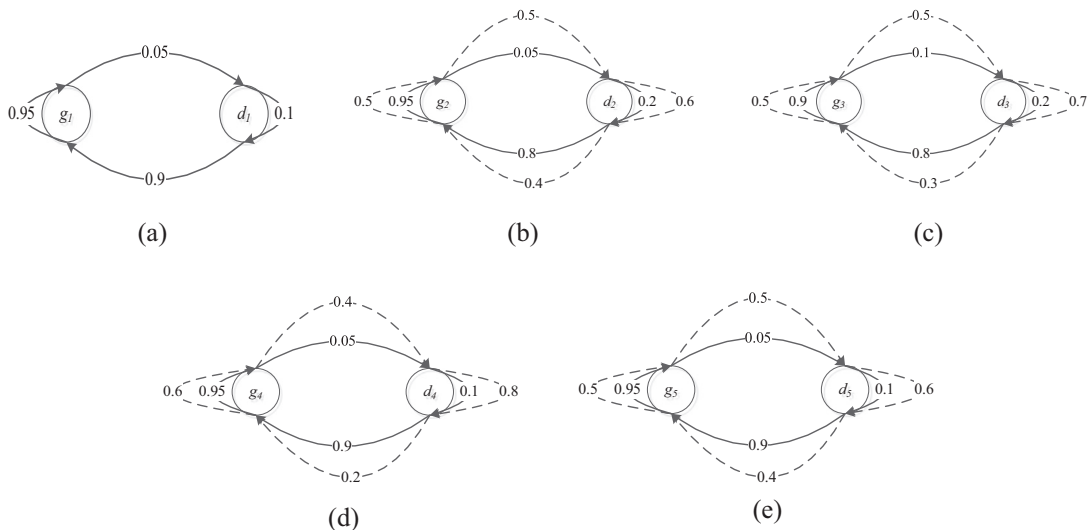


Fig. 8. Transition diagrams of the stages in the case.

failure probability with good coming parts  $\mu_3$ , which is the quality bottleneck parameter according to the definition. Proper changes of  $\mu_3$  can bring the largest improvement to  $P(g_3)$  and keep it from being a quality bottleneck stage.

### 5.3. Comparison with other models

#### 5.3.1. Comparison with the general Markov model

The first model we compare with is the general Markov model proposed by Li and Huang (2007) which ignores the quality of coming parts. In general Markov model, Li and Huang (2007) assumed that “each stage is independent” and “downstream stages could not correct defects introduced by upstream stages”. So in the general model, there are only two other than four parameters for each downstream stage: quality failure probabilities  $\lambda'_i$  and quality repair probabilities  $\mu'_i$ . In order to compare the general model with the proposed one, we first estimate parameters  $\lambda'_i$  and  $\mu'_i$  based on current parameters  $\alpha_1, \beta_1, \gamma_i, \mu_i, \eta_i$  and  $\theta_i$ . Below is the estimation procedure.

Step 1: Define  $M(i, j)$  as the quality state of product  $j$  after stage  $i$  ( $i = 1, 2, 3, 4, 5$ ).  $M(i, j)$  has two possible values 0 and 1, denoting defective and good states respectively.

Step 2: Randomly generate numbers within interval  $[0, 1]$ . Comparing these numbers with the corresponding transition probabilities and determine the value of  $M(i, j)$ . Taking the first stage as an example, if the quality of product  $j$  is good which means  $M(1, j) = 1$  and the generated number is larger than  $\alpha_1$ , then the next product  $j + 1$  is also good which implies  $M(1, j + 1) = 1$ . Similarly if  $M(1, j) = 0$  and the generated number is larger than  $1 - \beta_1$ , then  $M(1, j + 1) = 1$ .

Step 3: Let  $j$  increase from 1 to 10,000 to simulate the production process of 10,000 products. Record the values of  $M(i, j)$  and calculate the transition probabilities  $\lambda'_i$  and  $\mu'_i$  based on their definitions.  $\lambda'_i$  is the proportion that  $M(i, j)$  is good while  $M(i, j + 1)$  is defective.  $\mu'_i$  is the proportion that  $M(i, j)$  is defective while  $M(i, j + 1)$  is good.

Step 4: Repeat step 1–3 for 10,000 times and calculate the average values of  $\lambda'_i$  and  $\mu'_i$ .

Following this procedure, the quality failure probabilities  $\lambda'_i$  and quality repair probabilities  $\mu'_i$  are modified from  $\alpha_1, \beta_1, \gamma_i, \mu_i, \eta_i, \theta_i$ . Repeating the procedure for 10 times and the means and variances of  $\lambda'_i$  and  $\mu'_i$  are shown in Table 2.

Then  $\lambda'_i$  and  $\mu'_i$  are estimated as  $\lambda'_i = [0.05, 0.07, 0.13, 0.10, 0.09]$ ,  $\mu'_i = [0.90, 0.77, 0.74, 0.75, 0.81]$ . Under this circumstance, according to Eq. (3.34) in Li and Huang (2007), the probability of producing good product calculated by the general Markov

model is 70.6%, which shows a great difference with the actual final quality of 89.43%.

The result shows that for MMSs where the quality propagation exists, the general Markov model fails to describe its quality characteristics while the model proposed in this paper has higher accuracy on the evaluation of quality performance.

#### 5.3.2. Comparison with the modified quality flow model

From Section 2.5, we know that the Markov model proposed in this paper is quite different from the quality flow model developed by Ju et al. (2014). Despite of this, we try to do some comparisons between the results of them by adding two assumptions including:

- (1) In the quality flow model of Ju et al. (2014), the state of good state after repair is equivalent to the state of good state without repair.
- (2) In the Markov model proposed in this paper, the final quality is only related to the quality of the coming parts and has nothing to do with the state of stages.

Based on these assumptions, the estimation procedure is as follows.

Step 1 and Step 2 are same with those in Section 5.3.1.

Step 3: Let  $j$  increase from 1 to 10,000 to simulate the production process of 10,000 products. Record the values of  $M(i, j)$  and calculate the transition probabilities  $\lambda''_i, v''_i$  and  $\mu''_i$  based on their definitions.  $\lambda''_i$  and  $v''_i$  are the proportion that  $M(i, j)$  is good while  $M(i + 1, j)$  is defective.  $\mu''_i$  is the proportion that  $M(i, j)$  is defective while  $M(i + 1, j)$  is good.

Step 4: Repeat step 1–3 for 10,000 times and calculate the average values of  $\lambda''_i, v''_i$  and  $\mu''_i$ .

Following this procedure, we can estimate the parameters  $\lambda''_i, \mu''_i$  and  $v''_i$  needed in the quality flow model from  $\alpha_1, \beta_1, \gamma_i, \mu_i, \eta_i, \theta_i$  in the case. Repeating the procedure for 10 times and the means and variances of  $\lambda''_i (v''_i)$  and  $\mu''_i$  are shown in Table 3.

The results are  $\lambda''_i = v''_i = [0.05, 0.08, 0.15, 0.11, 0.10]$  and  $\mu''_i = [0.95, 0.89, 0.79, 0.84, 0.83]$ . Under this circumstance, according to Eq. (23) in Ju et al. (2014), the probabilities of producing good product estimated from the quality flow model is 89.02%, which is almost the same as the result from the proposed Markov model 89.07%. And it is also very close to the actual final quality 89.43%. So both the two models are highly accurate. But as stated in Section 2.5 and earlier in this section, the quality flow model cannot be applied to MMSs with RQIF directly and some assumptions must be added. This means the model proposed in this paper can be well applied to the MMSs with RQIF and keeps a high level of accuracy.

**Table 2**  
Means and variances of  $\lambda'_i$  and  $\mu'_i$ .

Number of experience	$\lambda'_1$	$\lambda'_2$	$\lambda'_3$	$\lambda'_4$	$\lambda'_5$	$\mu'_1$	$\mu'_2$	$\mu'_3$	$\mu'_4$	$\mu'_5$
1	0.0502	0.0726	0.1316	0.1004	0.0934	0.9031	0.7706	0.7415	0.7527	0.8124
2	0.0500	0.0732	0.1319	0.1005	0.0938	0.9004	0.7740	0.7411	0.7517	0.8126
3	0.0503	0.0737	0.1325	0.1010	0.0936	0.9013	0.7723	0.7412	0.7526	0.8101
4	0.0502	0.0727	0.1324	0.1004	0.0933	0.9033	0.7718	0.7422	0.7520	0.8126
5	0.0501	0.0731	0.1323	0.1002	0.0937	0.9022	0.7705	0.7421	0.7527	0.8101
6	0.0501	0.0732	0.1318	0.1001	0.0933	0.9040	0.7700	0.7401	0.7513	0.8105
7	0.0501	0.0731	0.1319	0.0999	0.0934	0.9029	0.7692	0.7411	0.7510	0.8129
8	0.0500	0.0732	0.1323	0.0998	0.0933	0.9018	0.7733	0.7441	0.7518	0.8117
9	0.0499	0.0734	0.1320	0.1001	0.0933	0.9017	0.7698	0.7401	0.7477	0.8111
10	0.0504	0.0735	0.1319	0.1002	0.0936	0.9013	0.7706	0.7418	0.7518	0.8098
Mean	0.0501	0.0732	0.1321	0.1003	0.0935	0.9022	0.7712	0.7415	0.7515	0.8114
Variance	2.2333E-08	1.1122E-07	8.7111E-08	1.1600E-07	3.5667E-08	1.2244E-06	2.4921E-06	1.3357E-06	2.1423E-06	1.4507E-06

**Table 3**  
Means and variances of  $\lambda_i''(v_i'')$  and  $\mu_i''$ .

Number of experience	$\lambda_1''(v_1'')$	$\lambda_2''(v_2'')$	$\lambda_3''(v_3'')$	$\lambda_4''(v_4'')$	$\lambda_5''(v_5'')$	$\mu_1''$	$\mu_2''$	$\mu_3''$	$\mu_4''$	$\mu_5''$
1	0.0526	0.0810	0.1490	0.1101	0.1013	0.9474	0.8940	0.7881	0.8382	0.8324
2	0.0526	0.0803	0.1485	0.1093	0.1017	0.9474	0.8926	0.7913	0.8376	0.8312
3	0.0526	0.0801	0.1486	0.1089	0.1006	0.9474	0.8918	0.7896	0.8389	0.8331
4	0.0526	0.0795	0.1491	0.1096	0.1010	0.9474	0.8935	0.7888	0.8383	0.8306
5	0.0526	0.0800	0.1488	0.1094	0.1010	0.9474	0.8940	0.7882	0.8388	0.8323
6	0.0526	0.0807	0.1494	0.1095	0.1013	0.9474	0.8924	0.7900	0.8381	0.8321
7	0.0526	0.0804	0.1491	0.1101	0.1019	0.9474	0.8927	0.7876	0.8384	0.8317
8	0.0526	0.0800	0.1492	0.1093	0.1006	0.9474	0.8933	0.7890	0.8392	0.8330
9	0.0526	0.0806	0.1491	0.1093	0.1011	0.9474	0.8938	0.7896	0.8396	0.8301
10	0.0526	0.0799	0.1488	0.1100	0.1014	0.9474	0.8906	0.7894	0.8376	0.8315
Mean	0.0526	0.08025	0.14896	0.10955	0.10119	0.9474	0.89287	0.78916	0.83847	0.8318
Variance	0.000E+00	1.939E-07	7.822E-08	1.606E-07	1.788E-07	0.000E+00	1.180E-06	1.152E-06	4.290E-07	9.578E-07

**Table 4**  
Comparison results of the three methods.

Methods	General Markov model (Li & Huang, 2007)	Quality flow model Ju et al. (2014)	Markov model of this paper
Basic system assumptions	(1) Stages are independent  (2) Only the states of stages are considered. The quality of coming parts is not considered	(1) Every stage has inspection and repair stations (2) Only the quality of coming parts is considered. The states of stages are not considered	(1) Stages are dependent and the inspection is at the end stage (2) Both the quality of coming parts and the states of stages are considered
Probabilities of producing good product	70.6%	89.02%	89.07%
Difference with the actual final quality	18.83%	0.41%	0.36%

All of the comparison results are listed in Table 4 and some conclusions can be drawn.

1. For the MMSs with RQIF, the general Markov model shows a low accuracy with an error of 18.83%. But the model proposed in this paper can well describe the quality characteristic of the system with an error of only 0.36%.
2. Although the quality flow model can be modified to estimate the final quality and has a small error of 0.41%, it is not proposed for MMSs with RQIF. It require that the system has a repair station after every stage. As stated in Section 2.5, the quality flow model fails to consider quality corrections by the system itself and cannot be applied to MMSs with RQIF directly.

In a word, the Markov model proposed in this paper are just meant for MMSs with RQIF and has high accuracy.

**6. Conclusion**

In this paper, a novel Markov model is developed to analyze quality propagation in MMSs with RQIF. In addition to deriving the analytical formulas to evaluate the final product quality, the monotonic properties are also investigated. It is shown that the monotonic properties of final quality in terms of quality failure probabilities and quality repair probabilities are not always the same as we expected. Based on the proposed model, a quality bottleneck identification method is explored. Finally, a case study is illustrated to show the effectiveness and practicability of the model. And it is also compared with the general Markov model and modified quality flow model, which shows that the proposed model is well applicable for MMSs with ROIF and has high accuracy. Further research opportunities can be focused on extending this proposed Markov model in MMSs which can produce multiple

types of products and enabling the model to be applicable for multi-product multi-stage manufacturing systems.

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**Appendix A**

**Proof of Eq. (12):**

From Eq. (10), we have

$$P(g_1g_2) = (1 - \alpha_1)(1 - \gamma_2)P(g_1g_2) + \beta_1(1 - \eta_2)P(d_1g_2) + (1 - \alpha_1)\mu_2P(g_1d_2) + \beta_1\theta_2P(d_1d_2) \tag{A.1}$$

$$P(d_1g_2) = \alpha_1(1 - \gamma_2)P(g_1g_2) + (1 - \beta_1)(1 - \eta_2)P(d_1g_2) + \alpha_1\mu_2P(g_1d_2) + (1 - \beta_1)\theta_2P(d_1d_2) \tag{A.2}$$

$$P(g_1d_2) = (1 - \alpha_1)\gamma_2P(g_1g_2) + \beta_1\eta_2P(d_1g_2) + (1 - \alpha_1)(1 - \mu_2)P(g_1d_2) + \beta_1(1 - \theta_2)P(d_1d_2) \tag{A.3}$$

$$P(d_1d_2) = \alpha_1\gamma_2(P(g_1g_2) + (1 - \beta_1)\eta_2P(d_1g_2) + \alpha_1(1 - \mu_2)P(g_1d_2) + (1 - \beta_1)(1 - \theta_2)P(d_1d_2)) \tag{A.4}$$

From Eq. (A.1), we have

$$P(d_1d_2) = \frac{1}{\beta_1\theta_2} \{ [1 - (1 - \alpha_1)(1 - \gamma_2)]P(g_1g_2) - \beta_1(1 - \eta_2)P(d_1g_2) - (1 - \alpha_1)\mu_2P(g_1d_2) \} \tag{A.5}$$

Substituting (A.5) into (A.2) leads to

$$P(g_1d_2) = \frac{\{[1 - (1 - \beta_1)(1 - \eta_2)]\beta_1\theta_2 + (1 - \beta_1)\theta_2\beta_1(1 - \eta_2)\}P(d_1g_2) - \{[1 - (1 - \alpha_1)(1 - \gamma_2)](1 - \beta_1)\theta_2 + \beta_1\theta_2\alpha_1(1 - \gamma_2)\}P(g_1g_2)}{\alpha_1\mu_2\beta_1\theta_2 - (1 - \alpha_1)\mu_2(1 - \beta_1)\theta_2} \quad (A.6)$$

Substituting (A.6) into (A.5), we obtain

$$P(d_1d_2) = \frac{\{(1 - \alpha_1)\mu_2\alpha_1(1 - \gamma_2) + \alpha_1\mu_2[1 - (1 - \alpha_1)(1 - \gamma_2)]\}P(g_1g_2) - \{(1 - \alpha_1)\mu_2\{[1 - (1 - \beta_1)(1 - \eta_2)] + \alpha_1\mu_2\beta_1(1 - \eta_2)\}\}P(d_1g_2)}{\alpha_1\mu_2\beta_1\theta_2 - (1 - \alpha_1)\mu_2(1 - \beta_1)\theta_2} \quad (A.7)$$

Rewriting (A.3), we have

$$[1 - (1 - \alpha_1)(1 - \mu_2)]P(g_1d_2) = (1 - \alpha_1)\gamma_2P(g_1g_2) + \beta_1\eta_2P(d_1g_2) + \beta_1(1 - \theta_2)P(d_1d_2) \quad (A.8)$$

Then substituting (A.6) and (A.7) into (A.8), we can solve  $P(d_1g_2)$  with respect to  $P(g_1g_2)$

$$P(d_1g_2) = \frac{W_2}{W_1}P(g_1g_2) \quad (A.9)$$

where

$$W_1 = [1 - (1 - \alpha_1)(1 - \mu_2)]\{[1 - (1 - \beta_1)(1 - \eta_2)]\beta_1\theta_2 + (1 - \beta_1)\theta_2\beta_1(1 - \eta_2)\} - \beta_1\eta_2[\alpha_1\mu_2\beta_1\theta_2 - (1 - \alpha_1)\mu_2(1 - \beta_1)\theta_2] + \beta_1(1 - \theta_2) \times \{(1 - \alpha_1)\mu_2[1 - (1 - \beta_1)(1 - \eta_2)] + \alpha_1\mu_2\beta_1(1 - \eta_2)\}$$

$$W_2 = (1 - \alpha_1)\gamma_2[\alpha_1\mu_2\beta_1\theta_2 - (1 - \alpha_1)\mu_2(1 - \beta_1)\theta_2] + \beta_1(1 - \theta_2)\{(1 - \alpha_1)\mu_2\alpha_1(1 - \gamma_2) + \alpha_1\mu_2[1 - (1 - \alpha_1)(1 - \gamma_2)]\} + [1 - (1 - \alpha_1)(1 - \mu_2)] \times \{[1 - (1 - \alpha_1)(1 - \gamma_2)](1 - \beta_1)\theta_2 + \beta_1\theta_2\alpha_1(1 - \gamma_2)\}$$

By substituting Eqs. (A.6) and (A.7) to Eq. (11), and combining with Eq. (A.9), we can solve the four steady-state probabilities  $P(g_1g_2)$ ,  $P(d_1g_2)$ ,  $P(g_1d_2)$ ,  $P(d_1d_2)$ .

According to Eq. (12), the final quality

$$P(g_2) = P(g_1g_2) + P(d_1g_2) = \frac{A}{A+B} \quad (A.10)$$

where

$$A = [\gamma_2(1 - \alpha_1) - \beta_1\eta_2][\alpha_1\mu_2\beta_1\theta_2 - \mu_2\theta_2(1 - \alpha_1)(1 - \beta_1)] + [1 - (1 - \beta_1)(1 - \eta_2) + \alpha_1(1 - \gamma_2)]\{\beta_1\theta_2[1 - (1 - \alpha_1)(1 - \mu_2)] + \beta_1\mu_2(1 - \theta_2)(1 - \alpha_1)\} + [1 - (1 - \alpha_1)(1 - \gamma_2)] + \beta_1(1 - \eta_2)\{\theta_2(1 - \beta_1)[1 - (1 - \alpha_1)(1 - \mu_2)] + \beta_1\alpha_1\mu_2(1 - \theta_2)\}$$

$$B = [\beta_1\theta_2 - \mu_2(1 - \alpha_1)]\{\gamma_2(1 - \alpha_1)[1 - (1 - \beta_1)(1 - \eta_2)] + \alpha_1\beta_1\eta_2(1 - \gamma_2)\} - [\alpha_1\mu_2 - \theta_2(1 - \beta_1)] \times \{\beta_1\eta_2[1 - (1 - \alpha_1)(1 - \gamma_2)] + \gamma_2\beta_1(1 - \alpha_1)(1 - \eta_2)\} + \{[1 - (1 - \alpha_1)(1 - \gamma_2)][1 - (1 - \beta_1)(1 - \eta_2)] - \alpha_1\beta_1(1 - \gamma_2)(1 - \eta_2)\}[1 - (1 - \alpha_1)(1 - \mu_2) + \beta_1(1 - \theta_2)] \quad \square$$

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